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# Monitoring the change in colour of meat: A comparison of traditional and kernel- based orthogonal transformations

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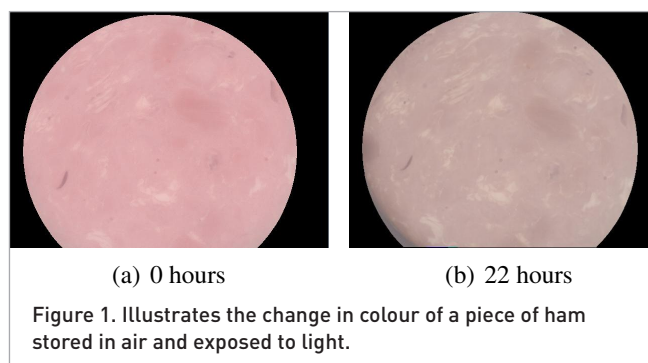
Currently, no objective method exists for estimating the rate of change in the colour of meat. Consequently, the purpose of this work is to develop a procedure capable of monitoring the change in colour of meat over time, environment and ingredients. This provides a useful tool to determine which storage environments and ingredients a manufacturer should add to meat to reduce the rate of change in colour. The procedure consists of taking multi-spectral images of a piece of meat as a function of time, clustering the pixels of these images into categories, including several types of meat, and extracting colour information from each category. The focus has primarily been on achieving an accurate categorisation since this is crucial to develop a useful method. The categorisation is done by applying an orthogonal transformation followed by *k*-means clustering. The purpose of the orthogonal transformation is to reduce the noise and amount of data while enhancing the difference between the categories. The orthogonal transformations principal components analysis, minimum noise fraction analysis and kernel-based versions of these have been applied to test which produce the most accurate categorisation.

**Keywords:** multi-spectral imaging, categorisation, principal components analysis (PCA), minimum noise fraction (MNF) analysis, kernel-based orthogonal transformations, *k*-means clustering

## Introduction

When a consumer shops for meat, it is often the colour and texture which determines how appetising the meat looks and thereby which piece of meat is purchased. The manufacturers desire a long shelf life for their products and therefore want to produce meat products which have as slow a rate of change in colour as possible. It is evident from Figure 1 that the colour changes rapidly if a piece of ham is stored in air and exposed to light. This implies that it is sensible to investigate which storage environments and additives are useful in achieving a slow rate of change.

To be able to both find useful additives and storage environments, as well as showing the effect, it is essential to have an objective method to estimate the change in colour. A manual



solution, which is the frame of reference for the developed method, is to let a sensory panel estimate the change of colour.

This solution could be made nearly objective by training the sensory panel thoroughly, but it would never be completely free of subjective opinions. Furthermore, it is expensive to train and test using a sensory panel.

The purpose of this work is therefore to develop an automated and so completely objective method which is able to monitor the development of the colour of meat as a function of time, environment and additives. This has been achieved by taking multi-spectral images of a piece of meat of interest at consecutive time points. The images are then categorised into several categories, including fat and different types of meat. Hereby the colour of each type of meat can be extracted for each time point. It is thereby possible to monitor the colour of the types of meat as time passes and evaluate the effects of additives and environment.

Focus has been on the categorisation, since this step is crucial in achieving an accurate estimation of the change in colour. The categorisation consists of two steps; an orthogonal transformation followed by  $k$ -means clustering<sup>1</sup> on a few of the most important factors of the transformation. The orthogonal transformation is applied to reduce the amount of data as well as to reduce the noise.

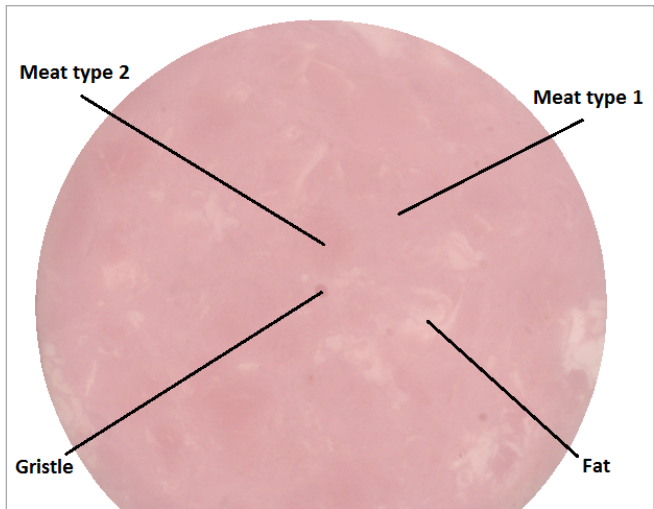
Several orthogonal transformations have been applied to investigate which is most suited for this application. The transformations are the traditional transformations principal components analysis (PCA)<sup>2</sup> and minimum noise fraction (MNF) analysis<sup>3</sup> as well as kernel-based versions of these transformations (kPCA<sup>4</sup> and kMNF<sup>5</sup>).

## Materials

The evaluated meat is a piece of ham consisting of reformed pork meat, but the procedure could be applied to any kind of meat. The ham is produced by adding brine to minced meat, mixing and then heat treating the mixture.<sup>6</sup> The final product contains two different types of meat, in addition to, for example fat and gristle, which comes from different mixes of muscle types. These two types of meat are the reason why a slice of ham is especially interesting to analyse since it is difficult to distinguish between these in the visible spectrum, which is illustrated in Figure 2. It is believed that the difference between these two types of meat is significant and estimating the change in colour of each of the types of meat is therefore of interest.

This work uses multi-spectral images of the pieces of ham as inputs to the analysis. The ham used to illustrate the usability of the developed method is kept in cold storage and multi-spectral images are taken regularly for 38 days. An example of such a multi-spectral image can be seen in Figure 3. The images are taken using Videometerlab,<sup>7</sup> developed and produced by Videometer A/S.

The procedure is to place a meat sample under an Ulbricht sphere (a sphere painted white on the inside giving diffuse backscattering of the light), which is illuminated with light-emitting diodes (LEDs) placed along the rim of the sphere. The



**Figure 2.** A pseudo RGB image of a piece of ham, i.e. an image constructed from the multi-spectral image such that it resembles a picture taken with a normal camera. This figure illustrates that the ham consists of several categories including fat, gristle and two types of meat. From this, it is evident that it is hard to distinguish between the two types of meat in the visible spectrum.

LEDs fire light pulses at specified wavelengths into the sphere to be reflected as scattered light onto the meat sample. This gives a uniform and reproducible illumination over a large area.

The multi-spectral images used here consist of  $n$  pixels where each pixel  $\mathbf{x}_i$  has  $p = 18$  variables corresponding to 18 colour bands. The data is collected in a data matrix  $\mathbf{X} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_n^T]^T$  where each row corresponds to a pixel and each column to a variable.  $\mathbf{X}$  therefore has size  $n \times p$ . Each band corresponds to a snapshot taken when the LEDs fire pulses with a certain wavelength. The wavelengths used range from ultraviolet (395 nm) to near infrared (970 nm) and they are listed in Table 1.

## Method

The procedure, to objectively analyse the change of colour of different types of meat in a piece of ham, is illustrated in Figure 4 and described in details in the following.

1. Take multi-spectral images of a piece of ham of interest at different points in time.

2. Do an orthogonal transformation of each of the multi-spectral images which results in a set of factors instead of the set of colour bands. This is done both to reduce the noise and the amount of data while maintaining the information.

3. Apply  $k$ -means clustering on a few of the most significant factors, thereby clustering the pixels of each image into four categories including the two types of meat.

4. Use the categorised images to extract the median colours (one for each colour band) of each of the categories from each

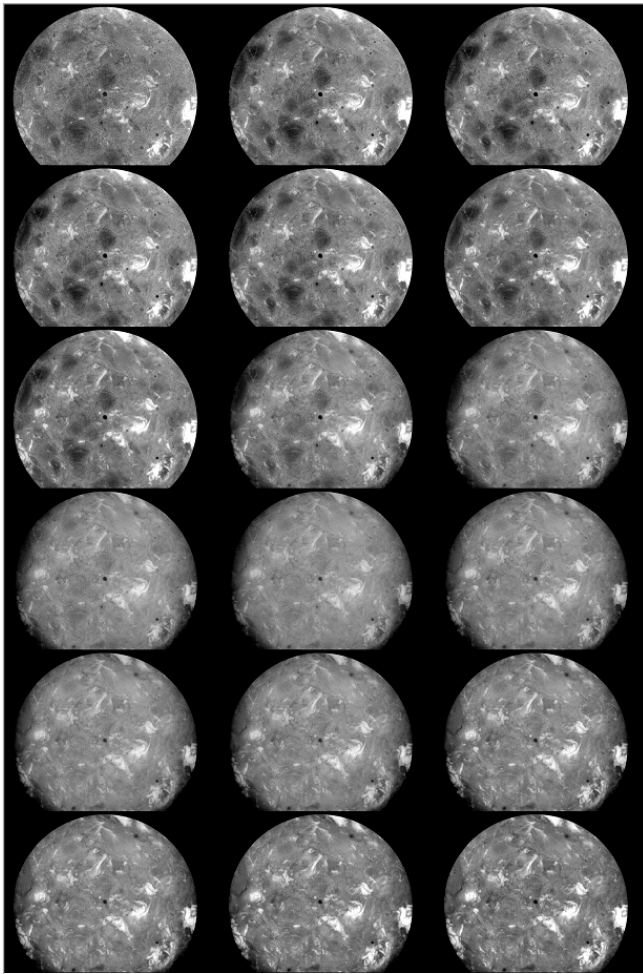


Figure 3. An example of a multi-spectral image of a piece of ham which consists of 18 colour bands. The wavelengths used to capture the spectral bands range from ultraviolet (395 nm) which is seen in the top-left corner to near infrared (970 nm) which is depicted in the bottom-right corner, where the wavelength is increasing from left to right and thereafter from top to bottom. A pseudo RGB image of this piece of ham can be seen in Figure 2.

of the multi-spectral images. Since the multi-spectral images have been taken at different points in time, the development of

Table 1. A table of the wavelength  $\lambda$  used to capture band  $p$  of the multi-spectral image.

$p$	1	2	3	4	5	6
$\lambda(\text{nm})$	395	430	450	470	505	565
$p$	7	8	9	10	11	12
$\lambda(\text{nm})$	590	630	645	660	700	850
$p$	13	14	15	16	17	18
$\lambda(\text{nm})$	870	890	910	940	950	970

the median colour values can be plotted as a function of time for each category. The median is used to reduce the influence of pixels which has been wrongly classified.

To reduce the amount of data while maintaining the information, an orthogonal transformation is applied. An obvious choice is to use PCA,<sup>2</sup> but the aim is also to reduce the noise which can be achieved by using a MNF<sup>3</sup> analysis. Furthermore, it is investigated whether applying kernel-based versions of PCA (kPCA)<sup>4</sup> and MNF (kMNF)<sup>5</sup> using a Gaussian kernel gives a better distinction between the categories.

After the transformation a relatively simple  $k$ -means clustering<sup>1</sup> is used to cluster the pixels in the image. The  $k$ -means clustering seeks to partition the  $n$  pixels  $\mathbf{X} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_n^T]^T$  in the image into  $k$  clusters  $C = \{C_1, C_2, \dots, C_k\}$ , called categories in this work, so as to minimise the within-cluster sum of squares

$$\arg \min_C \sum_{i=1}^k \sum_{x_j \in C_i} \|\mathbf{x}_j - \boldsymbol{\mu}_i\|^2$$

where  $\boldsymbol{\mu}_i$  is the mean of pixels in  $C_i$ .

## Traditional transformations

This section gives a brief introduction to the traditional orthogonal transformations PCA and MNF analysis. The transformations work on the pixels in the image  $\mathbf{X}$  where, the empirical mean has been subtracted from the data set to produce an  $\mathbf{X}$  with zero empirical mean.

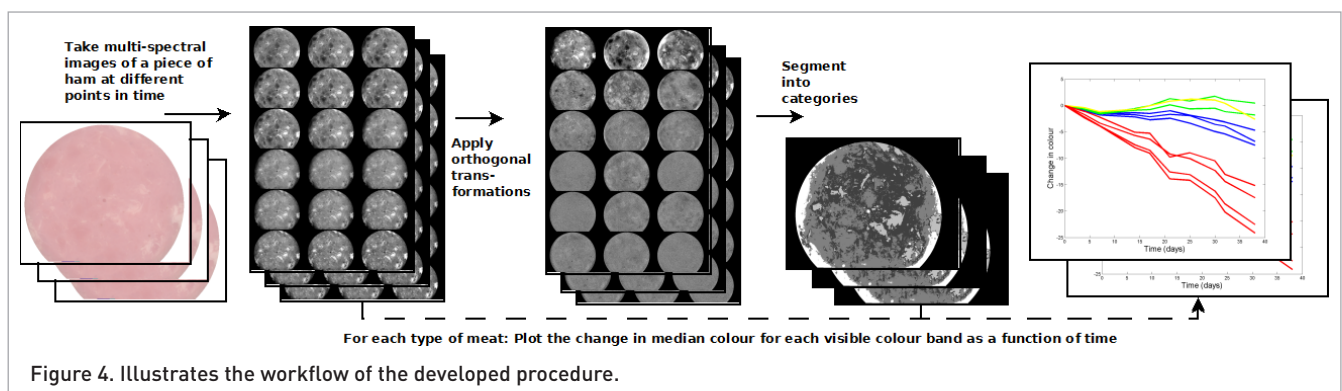


Figure 4. Illustrates the workflow of the developed procedure.



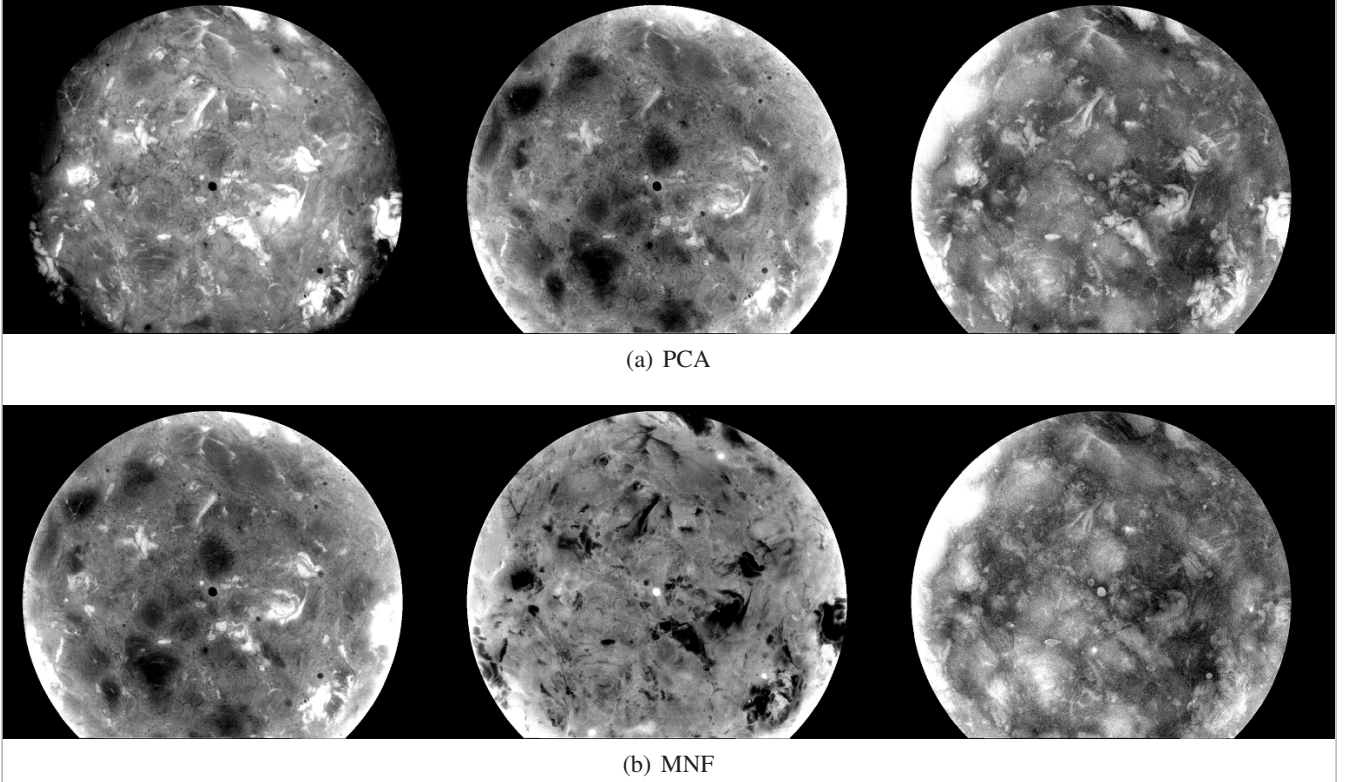


Figure 5. The first three factors which depict maximum variance or signal-to-noise ratio, produced by transforming the multi-spectral image from Figure 3 by the two transformations PCA and MNF. The choice to show only three factors of PCA, is due to the fact that most (>90%) of the variation is contained in these factors.

### Principal components analysis

The traditional PCA<sup>2</sup> is a linear transformation which maximises the variance of the data set  $\mathbf{X}$  by projecting the data onto a vector  $\mathbf{u}$  which is chosen such that the Rayleigh quotient

$$\lambda(\mathbf{u}) = \frac{\mathbf{u}^T \Sigma \mathbf{u}}{\mathbf{u}^T \mathbf{u}}.$$

is maximised. Here  $\Sigma$  is the variance–covariance matrix of the data set  $\mathbf{X}$ ;

$$\Sigma = \frac{\mathbf{X}^T \mathbf{X}}{n-1}.$$

This maximisation problem can be solved by solving the eigenvalue problem

$$\Sigma \mathbf{U} = \mathbf{U} \Lambda.$$

$\Sigma$  has the size  $p \times p$  (where  $p$  is the number of variables), which leads to  $p$  eigenvalues  $\lambda_i$  and corresponding mutually conjugate eigenvectors  $\mathbf{u}_i$ . The eigenvalues are gathered in the diagonal of the  $p \times p$  matrix  $\Lambda$ , while the eigenvectors are gathered in the columns of the  $p \times p$  matrix  $\mathbf{U}$ .

The principal components  $\mathbf{F}$  are then simply found by projecting the de-measured data  $\mathbf{X}$  onto the eigenvectors  $\mathbf{U}$

$$\mathbf{F} = \mathbf{X} \mathbf{U}.$$

The data have been transformed from the 18 spectral bands of the multi-spectral image  $\mathbf{X}$  into the 18 principal components  $\mathbf{F}$ , where the first three are displayed in Figure 5(a). The first principal component corresponds to maximum variance, whereas the following correspond to maximum variance under the condition that it is orthogonal to the preceding principal components.

### Minimum noise fraction

The MNF<sup>3</sup> analysis seeks to orthogonally transform the image  $\mathbf{X}$ , by a vector  $\mathbf{u}$ , such that it maximises the signal-to-noise ratio, i.e. maximises the image quality. This transformation therefore seeks to maximise

$$\frac{\text{Var}\{\mathbf{u}^T \mathbf{x}_S\}}{\text{Var}\{\mathbf{u}^T \mathbf{x}_N\}} = \frac{\mathbf{u}^T \Sigma \mathbf{u}}{\mathbf{u}^T \Sigma_N \mathbf{u}} - 1.$$

Here  $\mathbf{x}_S$  is a pixel in the image that corresponds to signal  $\mathbf{X}_S$ ,  $\mathbf{x}_N$  is a pixel in the image that corresponds to noise  $\mathbf{X}_N$ ,  $\Sigma$  is the variance–covariance matrix of  $\mathbf{X}$ , while  $\Sigma_N$  is the variance–covariance matrix of  $\mathbf{X}_N$ . Furthermore,  $\mathbf{X}_S$  and  $\mathbf{X}_N$  are assumed uncorrelated  $\Sigma = \Sigma_S + \Sigma_N$  and additive  $\mathbf{X} = \mathbf{X}_N + \mathbf{X}_S$ .  $\mathbf{X}_N$  is found by estimating the signal  $\mathbf{X}_S$  by smoothing  $\mathbf{X}$  and utilising that  $\mathbf{X}_N = \mathbf{X} - \mathbf{X}_S$ . Each pixel  $x_i \in \mathbf{X}_S$  is estimated by fitting a second order polynomial to a 5 pixels  $\times$  5 pixels neighbourhood of  $x_i$  and weighting this filter by a Gaussian weight.

The sought transformation can be found by maximising the Rayleigh quotient

$$\lambda(\mathbf{u}) = \frac{\mathbf{u}^T \Sigma \mathbf{u}}{\mathbf{u}^T \Sigma_N \mathbf{u}}.$$

The solution to this is the solution to a general eigenvalue problem

$$\Sigma \mathbf{U} = \Sigma_N \mathbf{U} \Lambda.$$

The de-means original data  $\mathbf{X}$  is then projected onto the eigenvectors  $\mathbf{U}$ , which results in the factors  $\mathbf{F}$

$$\mathbf{F} = \mathbf{X}\mathbf{U}.$$

An example of an MNF transformation on a multi-spectral image is seen in Figure 5(b).

## Kernel-based transformations

In this section, a description of kernel-based versions of the traditional orthogonal transformations PCA and MNF will be given as well as a brief introduction to kernel-based methods. For a more elaborate description of kernel-based methods and its theory, see Reference 8.

The kernel-based approach consists of two parts

- A mapping  $\phi$  of the data into a feature space using a kernel function  $\kappa$ .
- An analysis of the mapped data using a pattern analysis algorithm.

The idea is that it is possible to analyse the data using a *linear* pattern analysis algorithm after the mapping, which was not possible before. The approach is illustrated in Figure 6 for a classification problem.

### The kernel function

The pattern analysis algorithms are implemented such that only the pairwise inner products of the mapped data  $\phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$  are needed, which means it is not necessary to specify the mapping explicitly. These pairwise inner products can be efficiently computed from the original data using a kernel function  $\kappa(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$ . Examples of kernel functions are

Polynomial (homogeneous)

$$\kappa(\mathbf{x}_i, \mathbf{x}_j) = [\mathbf{x}_i^T \mathbf{x}_j]^d, \quad d \in \mathbb{N}, \quad d > 0.$$

Exponential

$$\kappa(\mathbf{x}_i, \mathbf{x}_j) = e^{-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{2\sigma^2}}.$$

Gaussian

$$\kappa(\mathbf{x}_i, \mathbf{x}_j) = e^{-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}}.$$

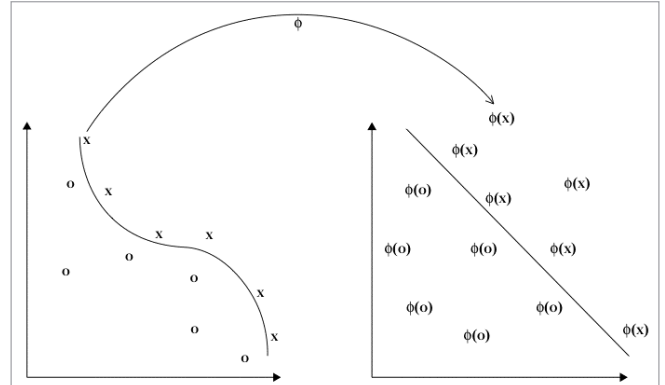


Figure 6. An illustration of a mapping of data into feature space where it is possible to use a linear analysis to separate the data into two classes which was not possible in the original space.<sup>8</sup> For illustration purposes the mapping is from two to two dimensions, but often the mapping is into a higher dimensional space.

The input parameters  $\mathbf{x}_i$  and  $\mathbf{x}_j$  are values of the original data, which are vectors of size  $p$ ,  $p \geq 1$ . Note that the distance measured between  $\mathbf{x}_i$  and  $\mathbf{x}_j$  is not fixed and can, for example, be chosen to be the  $L^1$  norm  $\|\mathbf{x}_i - \mathbf{x}_j\|$  or the  $L^2$  norm  $\|\mathbf{x}_i - \mathbf{x}_j\|_2$ .

A matrix called the kernel matrix  $\mathbf{K}$  which consists of elements  $\mathbf{K}_{ij} = \kappa(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$  is then constructed. The kernel matrix must be positive semi-definite and it is always symmetric since  $\kappa(\mathbf{x}_i, \mathbf{x}_j) = \kappa(\mathbf{x}_j, \mathbf{x}_i)$  for any kernel function. If  $n$  is again the number of data points, i.e. the number of pixels in the image, the size of the kernel matrix is  $n \times n$ .

This work uses a Gaussian kernel function since it is most commonly used in image analysis. Figure 7 illustrates how changing the  $\sigma$ -value, i.e. the standard deviation of the Gaussian curve, changes the result of a kMNF transformation. Throughout the rest of this paper, a  $\sigma$ -value of  $\sigma = 3 \cdot \sigma_0$ , where  $\sigma_0$  is the median distance between pixels in original feature space, is used since this seems to enhance the difference between the categories.

### The pattern analysis methods

When the kernel matrix  $\mathbf{K}$  is constructed the mapped data are analysed using a pattern analysis method. The pattern analysis methods in this case are the two eigenvalue-decomposition methods PCA and MNF, which are described below.

#### Kernel-based principal components analysis

The method used in traditional PCA, is called the primal method, whereas the method used in kernel-based principal components analysis (kPCA),<sup>4</sup> is called the dual method. The dual method is an analysis or eigenvalue decomposition, of the Gram matrix  $\mathbf{G} = \mathbf{X}\mathbf{X}^T$  instead of the variance-covariance matrix  $\Sigma = [1/(n-1)]\mathbf{X}^T\mathbf{X}$ . From a rewriting of the original eigenvalue problem, it is apparent that this method finds the same solution as PCA

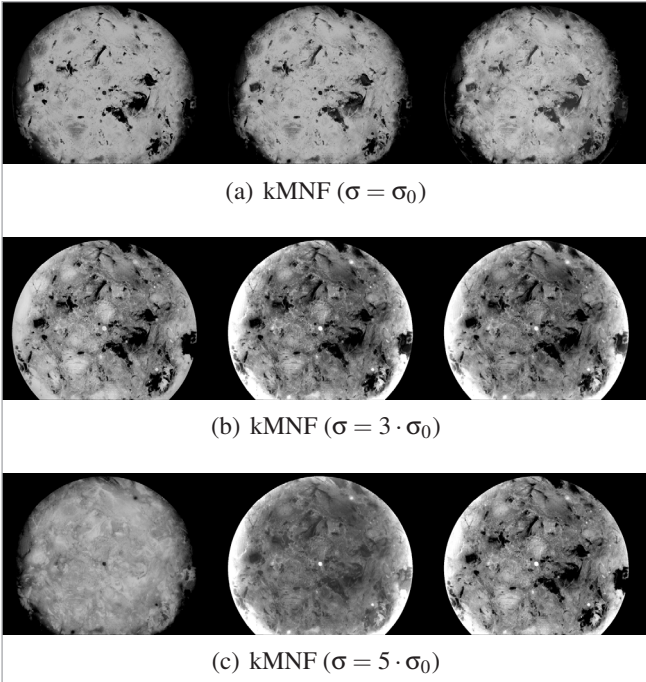


Figure 7. Illustrates the impact of using different  $\sigma$ -values when transforming the multi-spectral image in Figure 3 by a kMNF transformation using a Gaussian kernel function.

$$\begin{aligned}\frac{1}{n-1} \mathbf{X}^T \mathbf{X} \mathbf{U} &= \mathbf{U} \mathbf{\Lambda} \Leftrightarrow \\ \frac{1}{n-1} \mathbf{X} \mathbf{X}^T (\mathbf{X} \mathbf{U}) &= (\mathbf{X} \mathbf{U}) \mathbf{\Lambda} \Leftrightarrow \\ \frac{1}{n-1} \mathbf{X} \mathbf{X}^T \mathbf{V} &= \mathbf{V} \mathbf{\Lambda}.\end{aligned}$$

Here the eigenvectors  $\mathbf{u}_i$  in the columns of  $\mathbf{U}$  are a solution to the traditional PCA and the eigenvectors  $\mathbf{v}_i$  in the columns of  $\mathbf{V}$  solve the dual problem. These eigenvectors are related by  $\mathbf{U} = \mathbf{X}^T \mathbf{V} \mathbf{\Lambda}^{-1/2}$ , where  $\mathbf{\Lambda}^{-1/2}$  is a diagonal matrix with elements  $1/\sqrt{[(n-1)\lambda_i]}$ .

Instead of using the data set  $\mathbf{X}$  directly to compute the Gram matrix  $\mathbf{G} = \mathbf{X} \mathbf{X}^T$ , the data  $\mathbf{X}$  is mapped by the mapping  $\phi$  to  $\Phi = [\phi(\mathbf{x}_1)^T, \phi(\mathbf{x}_2)^T, \dots, \phi(\mathbf{x}_n)^T]^T$  before the matrix  $\mathbf{K}_{\mathbf{X}\mathbf{X}} = \Phi \Phi^T$  is computed. Consequently, the kernel matrix  $\mathbf{K}_{\mathbf{X}\mathbf{X}}$  contains the inner product of the mapped data. The eigenvalue problem is then given by

$$\frac{1}{n-1} \mathbf{K}_{\mathbf{X}\mathbf{X}} \mathbf{V} = \mathbf{V} \mathbf{\Lambda}$$

and the solution to the original problem is  $\mathbf{U} = \Phi^T \mathbf{V} \mathbf{\Lambda}^{-1/2}$ .

The principal components  $\mathbf{F}$  of the kPCA can be found in the same way as for the traditional methods, namely by projecting the data, which this time is mapped,  $\Phi$  onto the eigenvectors  $\mathbf{U}$

$$\mathbf{F} = \Phi \mathbf{U} = \Phi \Phi^T \mathbf{V} \mathbf{\Lambda}^{-1/2} = \mathbf{K}_{\mathbf{X}\mathbf{X}} \mathbf{V} \mathbf{\Lambda}^{-1/2}.$$

An example of the first three factors of a kPCA transformation applied to a multi-spectral image is seen in Figure 8(a).

### Kernel minimum noise fraction

To deduce the kernel-based MNF (kMNF) method,<sup>5</sup> the same procedure is used as for the kPCA transformation. The eigenvalue problem of the traditional MNF is rewritten to the dual problem, where Gram matrices are analysed instead of variance-covariance matrices

$$\frac{1}{n-1} \mathbf{X}^T \mathbf{X} \mathbf{U} = \frac{1}{n-1} \mathbf{X}_N^T \mathbf{X}_N \mathbf{U} \mathbf{\Lambda} \Leftrightarrow$$

$$\mathbf{X} \mathbf{X}^T \mathbf{X} \mathbf{U} = \mathbf{X} \mathbf{X}_N^T \mathbf{X}_N \mathbf{U} \mathbf{\Lambda} \Leftrightarrow$$

$$\mathbf{X} \mathbf{X}^T \mathbf{X} \mathbf{X}^T \mathbf{V} = \mathbf{X} \mathbf{X}_N^T \mathbf{X}_N \mathbf{X}^T \mathbf{V} \mathbf{\Lambda}.$$

The last step is made by replacing  $\mathbf{U}$  with  $\mathbf{X}^T \mathbf{V}$  on both sides.

The data  $\mathbf{X}$  and  $\mathbf{X}_N$  are then mapped to feature space using the mapping  $\phi$

$$\mathbf{X} \mathbf{X}^T \mathbf{X} \mathbf{X}^T \mathbf{V} = \mathbf{X} \mathbf{X}_N^T \mathbf{X}_N \mathbf{X}^T \mathbf{V} \mathbf{\Lambda} \Leftrightarrow$$

$$\Phi \Phi^T \Phi \Phi^T \mathbf{V} = \Phi \Phi_N^T \Phi_N \Phi^T \mathbf{V} \mathbf{\Lambda} \Leftrightarrow$$

$$\mathbf{K}_{\mathbf{X}\mathbf{X}}^2 \mathbf{V} = \mathbf{K}_{\mathbf{X}\mathbf{X}_N} \mathbf{K}_{\mathbf{X}\mathbf{X}_N}^T \mathbf{V} \mathbf{\Lambda}.$$

Here  $\mathbf{K}_{\mathbf{X}\mathbf{X}_N}$  is a matrix containing the inner products between the mapped original data  $\Phi$  and the mapped noise  $\Phi_N$ . The problem is a general eigenvalue problem and the eigenvectors  $\mathbf{V}$  are estimated. The original eigenvectors  $\mathbf{U}$  are then related to  $\mathbf{V}$  by  $\mathbf{U} = \Phi^T \mathbf{V}$ .

Again, the mapped data  $\Phi$  are projected onto the eigenvectors  $\mathbf{U}$  to find the factors  $\mathbf{F}$

$$\mathbf{F} = \Phi \mathbf{U} = \Phi \Phi^T \mathbf{V} = \mathbf{K}_{\mathbf{X}\mathbf{X}} \mathbf{V}.$$

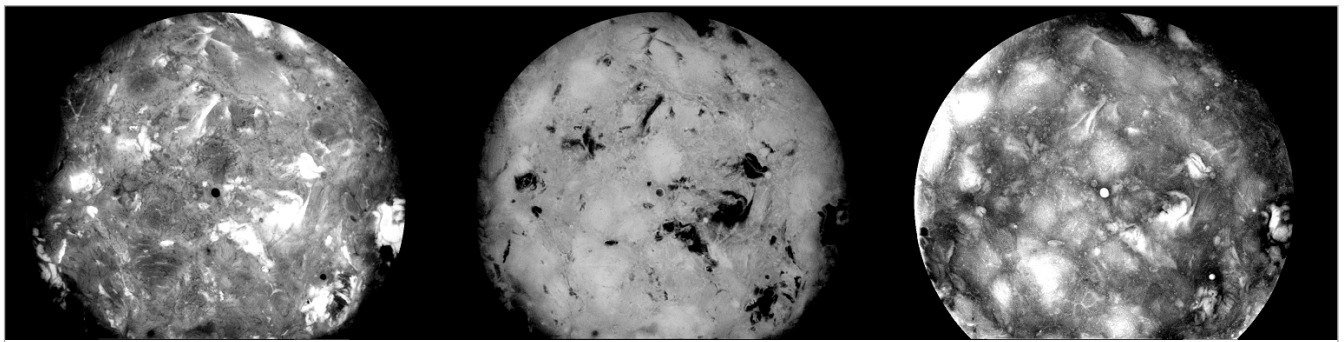
It is hereby shown how to perform a kMNF transformation, but it is also necessary to specify how the kernel matrix  $\mathbf{K}_{\mathbf{X}\mathbf{X}_N} = \Phi \Phi_N^T$  is constructed. This is an issue since the noise image  $\mathbf{X}_N$  is estimated using subtraction ( $\mathbf{X}_N = \mathbf{X} - \mathbf{X}_S$ ) and since a mapping is applied. This gives two possible ways to construct  $\Phi_N$ , namely estimate the noise and then map the noise to feature space,  $\phi(\mathbf{x}_N) = \phi(\mathbf{x} - \mathbf{x}_S)$ , or map the image and then estimate the noise in feature space  $\phi(\mathbf{x}_N) = \phi(\mathbf{x}) - \phi(\mathbf{x}_S)$ . The first option results in  $\mathbf{K}_{\mathbf{X}\mathbf{X}_N} = \mathbf{K}_{\mathbf{X}\mathbf{X}} - \mathbf{K}_{\mathbf{X}\mathbf{X}_S}$  and the second in  $\mathbf{K}_{\mathbf{X}\mathbf{X}_N} = \mathbf{K}_{\mathbf{X}\mathbf{X}} - \mathbf{K}_{\mathbf{X}\mathbf{X}_S}$ . Which one to choose is discussed by Gómez-Chova *et al.*<sup>9</sup> who find that it is correct to estimate the noise in kernel space, wherefore  $\mathbf{K}_{\mathbf{X}\mathbf{X}_N} = \mathbf{K}_{\mathbf{X}\mathbf{X}} - \mathbf{K}_{\mathbf{X}\mathbf{X}_S}$  is used in this work.

The first three factors of a transformation of kMNF are displayed in Figure 8(b).

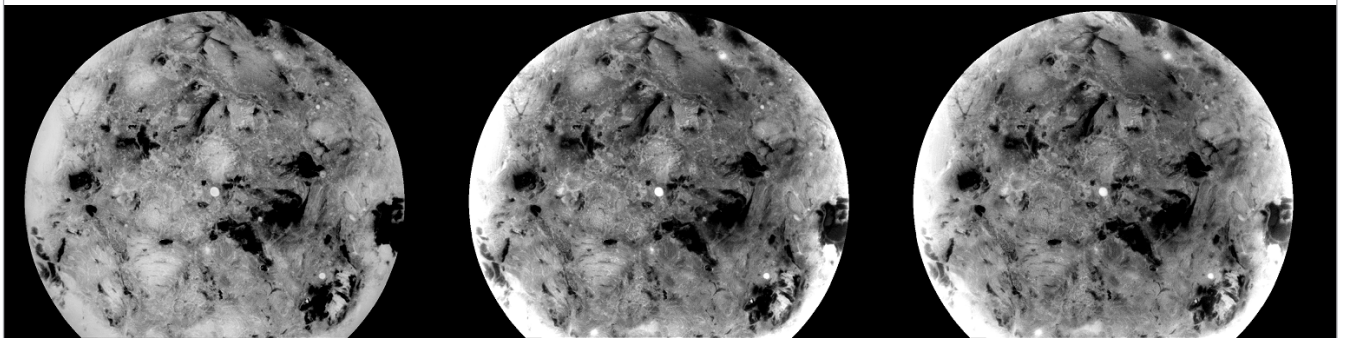
## Results

A piece of fresh ham has been chosen to illustrate the effects of the transformations and resulting categorisation. A multi-spectral image of this ham is shown in Figure 3 and a pseudo RGB image is found in Figure 2. The results of the PCA, MNF,



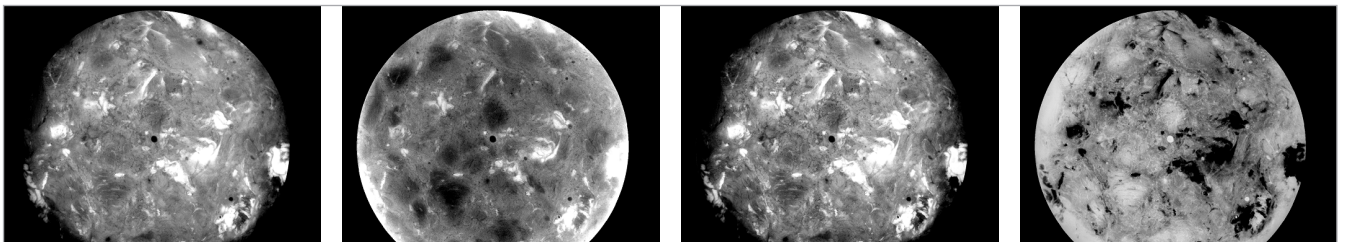


(a) kPCA



(b) kMNF

Figure 8. The first three factors which depict maximum variance or signal-to-noise ratio, produced by transforming the multi-spectral image from Figure 3 by the two kernel-based transformations kPCA and kMNF using a Gaussian kernel function with parameter  $\sigma = 3 \cdot \sigma_0$ , where  $\sigma_0$  is the median distance between pixels in original feature space.



(a) PCA

(b) MNF

(c) kPCA ( $\sigma = 3 \cdot \sigma_0$ )(d) kMNF ( $\sigma = 3 \cdot \sigma_0$ )

Figure 9. A comparison between the first factor of PCA, MNF, kPCA and kMNF transformations applied on the multi-spectral image seen in Figure 3. Both the kPCA and kMNF transformation uses a Gaussian kernel with  $\sigma = 3 \cdot \sigma_0$ .

kPCA and kMNF transformations on this image are seen in Figures 5 and 8 and the first factor of each of the transformations is seen in Figure 9. The four resulting categorisations, using the three first factors of each of the transformations, are seen in Figure 10.

To illustrate that the method works on other multi-spectral images, images of four pieces of ham, which have been kept in cold storage a different amount of time, are categorised and shown in Figure 11. In this case, the MNF transformation was used.

After categorising the multi-spectral images, it is possible to extract the median of each colour band and each type of meat from each image. A data set of 10 multi-spectral images, taken of pieces of ham which have been kept in cold storage

for up to 38 days, is used to create the graphs in Figure 12. The graphs depict the change in the median of each of the ten visible colour bands of meat type 1 [Figure 12(a)] and type meat type 2 [Figure 12(b)]. The change in each of the ten visible colour bands is depicted by a line in a colour corresponding to the wavelength. Only the visible colours are of interest since the purpose is to investigate how the customer sees the change in colour.

## Discussion

In this section, the results of the four transformations will be compared and their advantages and disadvantages will be



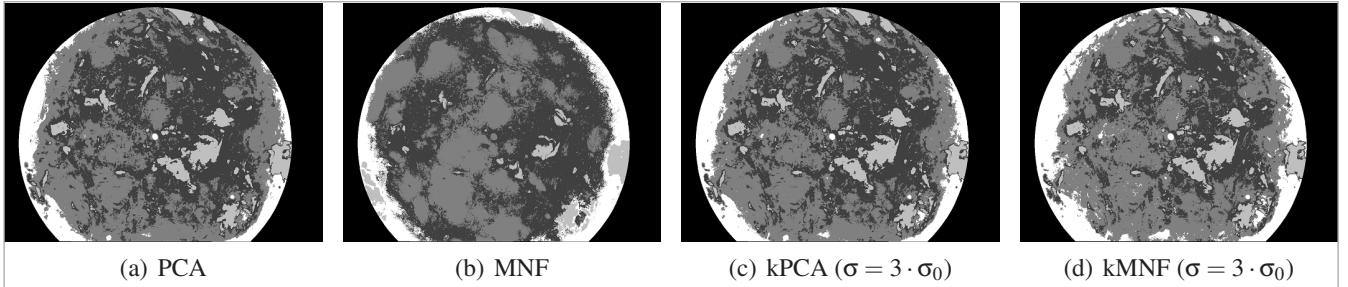


Figure 10. The categories of the multi-spectral image from Figure 3 using PCA, MNF, kPCA or kMNF followed by *k*-means clustering. The *k*-means clustering algorithm uses the first three factors of the transformations (seen in Figures 5 and 8) and divides the pixels into four categories. The four categories corresponds to meat type 1 (dark grey), meat type 2 (grey), fat (light grey) and other (white).

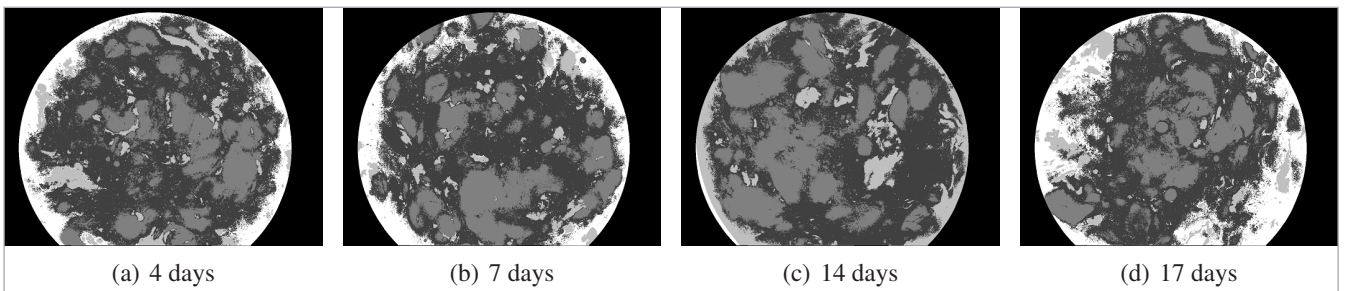


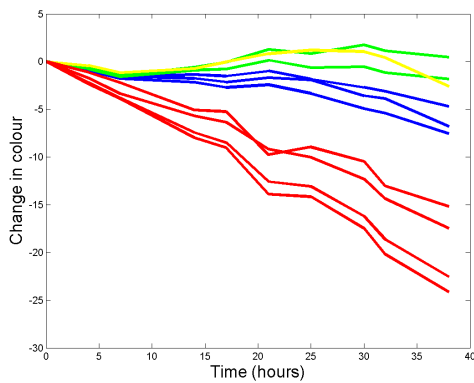
Figure 11. The categories [meat type 1 (dark grey), meat type 2 (grey), fat (light grey) and other (white)] extracted from four different multi-spectral images using the developed method which utilises an MNF transformation. The four multi-spectral images on which the method was applied depict ham which have been kept in cold storage for the duration depicted in the caption.

discussed. Furthermore, the applicability of the procedure will be evaluated.

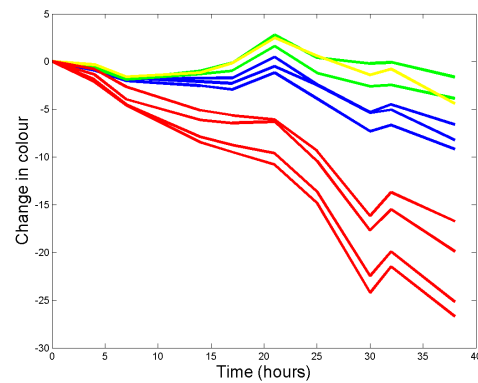
The traditional orthogonal transformations have several advantages compared to the kernel-based ones. They are well established and widely used, which means many people know them and more importantly know how to use them. Furthermore, they are relatively fast and have a small memory consumption. The latter is especially important when working with images, since imaging often produces a large amount of data.

The data matrix  $\mathbf{X}$  has size  $n \times p$ , i.e. number of pixels in the image times number of colour bands. The variance-covariance matrix  $\Sigma$  used in the traditional transformations is therefore of size  $p \times p$ , which in the case presented here is  $18 \times 18$ . The Gram matrix on the other hand is of size  $n \times n$ , which means it is a huge matrix with a size of the order millions times millions. This results in infeasible memory consumption, when using kernel-based transformations.

The solution to the problem adopted here is random sub-sampling. This means a subset of the data is randomly chosen

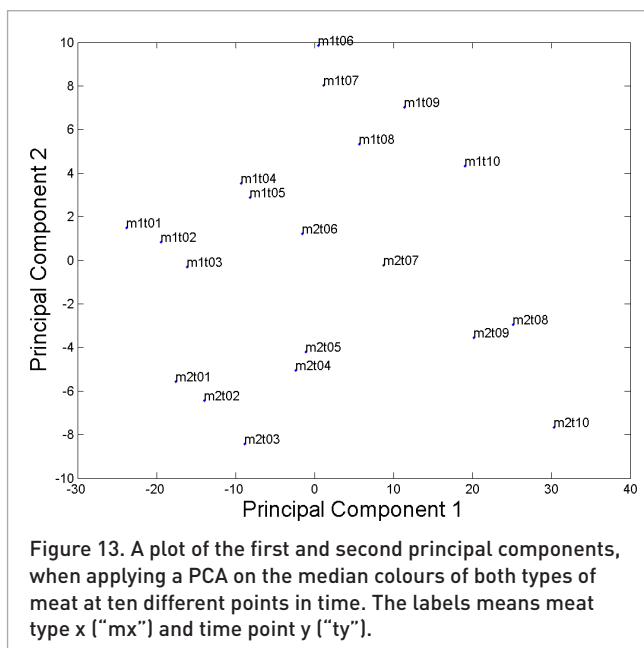


(a) Meat type 1



(b) Meat type 2

Figure 12. Displays the change in the median colour of each of the two types of meat for each of the visible wavelengths. The colour of the line corresponds to the colour of the light used to capture the image.



for the construction of the kernel matrices. The data set  $\mathbf{X}$  is therefore reduced to the size of  $s \times p$ , where  $s$  is the number of samples, before constructing the kernel matrix  $\mathbf{K}_{\mathbf{xx}}$ , which then has size  $s \times s$ . This approach will reduce the memory consumption to a tolerable level, but will also introduce inconsistency, i.e. the result will potentially be different when the transformation is applied to other training samples.

Another solution to the problem of memory consumption would be to use an iterative method which is slower but uses less memory. Using this method, it is possible to use all the pixels to create the transformation which results in consistency. An iterative version of kPCA has been developed by Kim *et al.*,<sup>10</sup> while the development of iterative versions of kMNF is left for future research.

The advantage of the kernel-based transformations is their ability to capture non-linear tendencies and enhance details which are not possible to enhance when using traditional transformations. This ability is due to the mapping of the data, where it is possible to choose the kernel function and fine tune the parameter to fit the patterns of interest.

The advantage of the traditional transformations in terms of memory consumption, suggests that the kernel-based transformations should be used only when they give a considerably better result. In this case, a better result would be a substantially better separation between the categories and thereby a better categorisation. This is not the case as illustrated by Figures 9 and 10, where it is seen that differences between kPCA, kMNF and PCA are not conspicuous. This suggests that the possible gain of using kernel-based methods in this case is not substantial enough to cope with the memory consumption and resulting inconsistency.

The difference between PCA and MNF is on the other hand considerable and it is evident from the smooth categorisation in Figure 11 that MNF greatly reduces the noise. That MNF removes noise and not signal is suggested by the resulting

categorisation which has been identified by an expert to resemble an accurate categorisation. Furthermore, the categorisation achieved by the method using MNF consists of clustered regions, which is also expected since the meat, fat etc. is expected to be clustered.

In Figure 12, it is seen that, especially for wavelengths in the red part of the spectrum, the reflection of light decreases significantly as a function of time for both types of meat, which was also expected. This shows that it is possible to monitor the change in colour of each type of meat as a function of time using the described method.

A PCA has been applied to the median colours of the two types of meat which has been sampled at ten points in time and each sample consists of 18 spectral bands. The result can be seen in Figure 13, where it is evident that the first principal component corresponds to time. It is therefore shown that the maximum variance in the colour is due to development in time and furthermore, it is seen from the coefficient of the first principal component, that the variance is primarily found in the four red spectral bands. The second principal component shows that there is a difference between the two types of meat and that the difference is significant in all the visible bands.

## Conclusion

An automatic and thereby objective method, for monitoring the change in colour as a function of time in several types of meat, has successfully been developed. The method was tested on multi-spectral images of pieces of ham which were kept cold for up to 38 days and showed great promise with the expected result; the colour of the red bands is decreasing over time.

Therefore, this method shows great promise to be able to objectively compare the rates of change in colour of ham samples which have been kept under different storage conditions or contain different additives. It would thereby be possible to determine the effect of environment or additives on the rate of change in the colour of meat and thereby selecting the optimal environment and additives.

The main focus has been to cluster pixels in multi-spectral images into categories representing the two types of meat, fat and other. This step is crucial to be able to monitor each type of meat and thereby get an accurate estimate of their change in colour. The orthogonal transformations PCA, MNF, kPCA and kMNF were applied to find the one which both reduces the noise and the amount of data, while enhancing the difference between the categories.

From the achieved results, it can be concluded that kernel-based transformations do not give a considerably greater difference between categories or reduction of noise compared to PCA. The kernel-based methods are therefore not an advantage in this case, especially not when considering their disadvantages of, for example, huge memory consumption. The results also show that the MNF transformation both reduces the noise and amount of data while keeping the information, while PCA only achieves the latter. Furthermore, using MNF in conjunction with  $k$ -means clustering results in accurate categorisations according to an expert.

It was also shown that difference in colour between the two types of meat is significant in the visible spectrum. It was also shown that the change in colour is focused in the red spectrum. These results suggest that the categorisation into the two types of meat is necessary, if one wants to achieve accurate results in the entire visible spectrum.

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